SEMESTER S7

TOPICS IN THEORETICAL COMPUTER SCIENCE

Course Code	**PECST795**	**CIE Marks**	40
Teaching Hours/Week (L: T:P: R)	3:0:0:0	**ESE Marks**	60
Credits	5/3	**Exam Hours**	2 Hrs. 30 Min.
Prerequisites (if any)	PCCST303 PCCST502	**Course Type**	Theory

Course Objectives:

1. To understand and apply spectral graph theory techniques to analyze and solve complex graph problems, such as community detection and network design, through detailed study and hands-on assignments.
2. To develop and evaluate LP- and SDP-based approximation algorithms for NP-hard problems, including real-world applications like scheduling and optimization, by implementing these algorithms and assessing their performance in practical scenarios

SYLLABUS

Module No.	Syllabus Description	Contact Hours
1	Spectral Graph Theory - Introduction to Spectral Graph Theory, Graph Laplacians: Definition and Properties, Eigenvalues and Eigenvectors of Laplacian matrices, Cheeger's Inequality, Graph Partitioning. **Assignments:** 1. Implement Cheeger's inequality for a set of sample graphs. Compare the theoretical results with empirical data to analyze the effectiveness of different partitioning algorithms. Use a set of sample graphs such as Erdős-Rényi Random Graphs, Barabási-Albert Model: Known for scale-free properties, and Regular Graphs. Compare theoretical results with empirical data using different partitioning algorithms such as Spectral Clustering - Uses the eigenvectors of the Laplacian matrix, K-means Clustering - Applied to spectral embeddings of the graph, Normalized Cut - Minimizes the normalized cut criterion. Measure how close the empirical conductance is to the theoretical lower bound provided by Cheeger's inequality. Analyze which algorithms produce cuts with conductance values closer to the	9

	theoretical bounds. Real-world Application: Apply Cheeger's inequality to social network analysis to detect community structures. 2. Analyze the properties of the Laplacian matrix of a given graph (Erdős-Rényi Random Graphs). Compute its eigenvalues and eigenvectors and discuss the implications for graph partitioning. Examine the use of graph Laplacians in network community detection.	
2	Spectral Clustering - Introduction to Clustering and Spectral Clustering, Normalized Cut, Eigenvalue Techniques for Clustering, Spectral Clustering Algorithm, Applications of Spectral Clustering. **Assignment:** 1. Implement a spectral clustering algorithm and apply it to a real-world dataset (Iris dataset). After running the spectral clustering algorithm, evaluate the results using metrics such as Silhouette Score and Adjusted Rand Index (ARI). Plot the data points colored by their cluster assignments to visually inspect the clustering. Compare spectral clustering with other clustering techniques (e.g., k-means, hierarchical clustering) on the three types of datasets - Synthetic Data, Real-World Data (Iris Dataset), and High-Dimensional Data (Text Data (Use TF-IDF features)). Discuss the advantages and limitations of spectral clustering in different scenarios. Real-world Application: Use clustering results for anomaly detection in network security.	**9**
3	Expanders - Introduction to Expander Graphs, Properties and Construction of Expanders, edge-expanders, vertex-expanders, spectral-expanders, Expander Mixing Lemma, Random walks on expanders graphs, Applications of Expander Graphs: Error-Correcting Codes. **Assignments:** 1. Study the construction and properties of expander graphs such as Erdős-Rényi graphs, Ramanujan graphs and Cayley graphs. Implement algorithms for generating expander graphs and analyze their properties based on spectral gap and expansion property. 2. Apply expander graphs to error-correcting codes. Design and test codes based on expanders, and evaluate their performance in terms of error correction capabilities. Simulate a communication channel with	**9**

	added noise and measure the performance of the expander code in correcting errors. Evaluate the BER, code rate, and error correction capability by comparing the number of errors corrected versus the total number of errors introduced.	
4	LP- and SDP-based Approximation Algorithms for NP-Hard Problems - Linear Programming (LP) Relaxations and their Use in Approximation: Vertex Cover and Set Cover, Semidefinite Programming (SDP) and its Applications: Max-Cut Problem. **Assignments:** 1. Implement and evaluate LP relaxations for vertex cover and set cover problems (use Erdős-Rényi Graphs). Compare the results with exact solutions and analyze the quality of the approximations. 2. Develop and test approximation algorithms for Max-cut problem using SDP relaxations. Assess the performance and efficiency of your algorithms on various datasets. To assess the performance and efficiency of the SDP-based Max-Cut approximation, test the algorithm on various types of graphs, including: Erdős-Rényi Graphs, Barabási-Albert Graphs, and Real-world Graphs. Compare the cut values obtained from the SDP relaxation and rounding with known or exact solutions if available. For large graphs, use heuristics or bounds for comparison. Measure the time taken to solve the SDP relaxation and perform the rounding. This includes the time for solving the SDP problem and the time for eigen-decomposition.	**9**

Course Assessment Method
(CIE: 40 marks, ESE: 60 marks)

Continuous Internal Evaluation Marks (CIE):

Attendance	*Internal Ex*	*Evaluate*	*Analyse*	*Total*
5	**15**	**10**	**10**	**40**

Criteria for Evaluation(Evaluate and Analyse): 20 marks

Assignment evaluation pattern:

- Theoretical Understanding (25%) - Evaluate the clarity and accuracy with which theoretical concepts such as spectral graph theory, clustering algorithms, expanders, and approximation methods are explained and applied.
- Application of Theory (25%) - Assess how well the theoretical methods are applied to address assignment problems. Check if solutions are relevant, accurate, and demonstrate a good grasp of the theoretical background.
- Depth of Analysis (25%) - Analyze the depth of the problem analysis, including how well the assignment tackles complex aspects and nuances of the problem.
- Interpretation of Results (25%) - Evaluate the meaningfulness and relevance of the conclusions drawn from the analysis. Check if the results provide significant insights into the problem**.**

End Semester Examination Marks (ESE):

In Part A, all questions need to be answered and in Part B, each student can choose any one full question out of two questions

Part A	Part B	Total
• 2 Questions from each module. • Total of 8 Questions, each carrying 3 marks **(8x3 =24 marks)**	• 2 questions will be given from each module, out of which 1 question should be answered. • Each question can have a maximum of 3 subdivisions. • Each question carries 9 marks. **(4x9 = 36 marks)**	**60**

Course Outcomes (COs)

At the end of the course students should be able to:

Course Outcome		Bloom's Knowledge Level (KL)
CO1	Understand and explain fundamental concepts of Spectral Graph Theory, including Laplacian matrices and their applications.	**K2**
CO2	Apply spectral clustering techniques to real-world data and evaluate clustering performance using appropriate metrics.	**K5**
CO3	Construct and analyze expander graphs, and assess their applications in network design and error-correcting codes.	**K4**
CO4	Develop and implement LP- and SDP-based approximation algorithms for solving NP-Hard problems, and compare their performance.	**K5**
CO5	Demonstrate the ability to solve complex theoretical problems using advanced algorithms and techniques covered in the course.	**K4**

Note*: K1- Remember, K2- Understand, K3- Apply, K4- Analyse, K5- Evaluate, K6- Create*

CO-PO Mapping Table (Mapping of Course Outcomes to Program Outcomes)

	PO1	PO2	PO3	PO4	PO5	PO6	PO7	PO8	PO9	PO10	PO11	PO12
CO1	3	3	3									3
CO2	3	3	3									3
CO3	3	3	3		3							3
CO4	3	3	3	3								3
CO5	3	3	3	3	3							3

Note: *1: Slight (Low), 2: Moderate (Medium), 3: Substantial (High), -: No Correlation*

Text Books				
Sl. No	**Title of the Book**	**Name of the Author/s**	**Name of the Publisher**	**Edition and Year**
1	Spectral Graph Theory (CBMS Regional Conference Series)	Fan R. K. Chung	American Mathematical Society	1/e, 1997
2	Algebraic Graph Theory	Norman Biggs	Cambridge India	2/e, 2016
3	Approximation Algorithms	Vijay V. Vazirani	Springer Nature	2/e, 2013
4	Convex Optimization	Stephen Boyd, Lieven Vandenberghe	Cambridge University Press	1/e, 2004

Reference Books				
Sl. No	**Title of the Book**	**Name of the Author/s**	**Name of the Publisher**	**Edition and Year**
1	Algebraic Graph Theory	C. Godsil, G.F. Royle	Springer Nature	1/e, 2009
2	The design of approximation algorithms	David Williamson, David Shmoys	Cambridge University Press	1/e, 2011
3	Randomized Algorithms	Rajeev Motwani, Prabhakar Raghavan	Cambridge University Press	1/e, 2004
4	Probability and Computing: Randomization and Probabilistic Techniques in Algorithms and Data Analysis	Michael Mitzenmacher, Eli Upfal	Cambridge University Press	3/e, 2017
5	Graph Theory and Complex Networks: An Introduction	Maarten Van Steen	Maarten Van Steen	1/e, 2010

Video Links (NPTEL, SWAYAM…)	
No.	**Link ID**
1	https://archive.nptel.ac.in/courses/128/106/128106001/